

ZERO-METALLICITY STARS AND THE EFFECTS OF THE FIRST STARS ON REIONIZATION

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ABSTRACT

We present stellar structure and atmosphere models of metal-free stars and examine them from a cosmological point of view. Metal-free stars exhibit high effective temperatures and small sizes relative to metal-enriched stars of equal mass. These unique physical characteristics enhance the ionizing photon production by metal-free stars, particularly in the He II ($h\nu \geq 4$ Ryd) continuum. The star formation rate of metal-free stars necessary to reionize the hydrogen in the universe by $z = 5$ is consistent with the inferred star formation rate at that epoch. However, the hard stellar spectra are inconsistent with the observations of He II opacity in the IGM at $z \sim 3$, indicating that the period of metal-free star formation ended before that epoch. We examine the effects of these stars on the ionization balance of the IGM, the radiative feedback of the first luminous objects, and the extragalactic radiation field. We comment on the prospects for detecting metal-free stellar populations with the $\lambda 1640$ and $\lambda 4686$ recombination lines of He II.

Subject headings: stars: early-type — intergalactic medium — cosmology: theory

1. INTRODUCTION

We know from observational constraints that the hydrogen in the intergalactic medium (IGM) was reionized by redshift $z \sim 5$ (Schneider, Schmidt, & Gunn 1991) and that the He II was reionized at $z \sim 3$ (Reimers et al. 1997). However, the exact nature of the ionizing sources is still uncertain. The observed drop in the space density of bright quasars at $z \geq 3$ (Pei 1995) suggests that early stellar populations played a role in the reionization of hydrogen, but it is not known whether hot stars produced photons at rates sufficient to ionize the universe before this epoch.

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Our understanding of reionization is closely connected to our knowledge of the extragalactic radiation field. Models of reionization assume an extragalactic spectrum composed of the distinct intrinsic spectra of active galactic nuclei (AGN) and star-forming galaxies. This composite spectrum determines the reionization epoch, the He II/H I ionization ratio, and metal-line absorption ratios at $z < 5$. Models by Giroux & Shull (1997) of the observed Si IV/C IV ratio at $z \sim 3$ (Songaila & Cowie 1996) and subsequent work by Haardt & Madau (1996) and Fardal et al. (1998) on the He II Gunn-Peterson effect demonstrated that the observations are consistent with an extragalactic spectrum produced by a mixture of QSOs and hot stars. These conclusions depend, however, on the assumed shape of the ionizing spectrum of stars. While the details of these spectra vary (Sutherland & Shull 1999; Leitherer et al. 1999), one common element is the assumption that hot stars contribute few He II ionizing photons.

Some models of reionization assume a phenomenological prescription for star formation, in which a single parameter describes the conversion efficiency of mass to stars and then to ionizing photons (Gnedin & Ostriker 1997). Others use existing model grids of stellar structure and atmospheres designed for application to metal-poor environments (Haiman & Loeb 1997). The first method ignores the details of star formation, ionizing photon production, and radiation escape from the immediate regions of star formation. The second method has the drawback of applying theoretical calculations of metal-poor stars to the very different regime of zero metallicity. The existing grids of stellar evolution tracks extend to $Z = 0.001$ (Schaerer et al. 1996 and references therein). Existing model atmospheres extend to $Z = 2 \times 10^{-7}$ but are limited in the range of stellar parameters (Kurucz 1992). These models are meant for application to low-metallicity starbursts (Leitherer et al. 1999) and metal-poor galactic halo populations. As shown by existing models of metal-free stars (Ezer 1972; Ezer & Cameron 1971; El Eid et al. 1983), however, stars with $Z \sim 0.001$ are quite different from their $Z = 0$ counterparts. Thus, when we consider the effects of stellar populations on reionization, we must use true metal-free models to predict the ionizing photon production of the first generation of stars.

In this *Letter* we adopt the common term “Population III” or “Pop III” for metal-free stars, which are understood to have formed from primeval gas. Although extremely metal-poor populations ($Z \lesssim 0.001$) may fit an observer’s definition of Pop III, we apply that label to metal-free stars only. In § 2 we present structure and atmosphere models of metal-free stars, and in § 3 we predict ionizing photon yields from these model stars. In § 4 we evaluate the effects of these models on the epoch of reionization, and in § 5 we comment on further cosmological implications of metal-free stellar populations.

2. STRUCTURE AND ATMOSPHERE MODELS

The models presented here are static stellar structure models calculated using a fitting-method technique that incorporates OPAL radiative opacities (Rogers & Iglesias 1992) and analytic expressions for energy generation. These models were used to predict the effective temperature

(T_{eff}), luminosity (L), and surface gravity (g) of stars with mass $2 - 90 M_{\odot}$ (at $5 - 10 M_{\odot}$ intervals). There is currently no full set of evolutionary tracks for metal-free stars, and our models do not incorporate evolution. However, the existing evolutionary tracks for metal-free stars (Castellani, Chieffi, & Tornambe 1983; Chieffi & Tornambe 1984) show that, like their metal-enriched counterparts, these stars become systematically cooler, larger, and more luminous over their H-burning lifetimes, which are similar in duration. Therefore, we assume that metal-free tracks differ in their first-order characteristics only in their starting point on the Hertzsprung-Russell (HR) diagram. If so, the “gain” in ionizing photons at $Z = 0$ is maintained throughout the main sequence (MS) lifetime of the star, when most of its ionizing radiation is released.

Existing evolutionary tracks of metal-free stars show that these stars may build up a small fraction of C nuclei ($Z_C \sim 10^{-10}$) via the triple- α process before they join the H-burning main sequence (El Eid et al. 1983; Castellani et al. 1983). Following this result, we assume that stars with $M \geq 15 M_{\odot}$ are enriched to $Z_C = 10^{-10}$ via triple- α burning prior to the onset of MS H-burning. We use these pre-enriched models in all our analysis. We will explore the detailed effects of pre-MS self-enrichment in a later paper, once a comprehensive grid of tracks is available.

Figure 1 shows an HR diagram for zero-age main sequence models with Pop I and Pop III metallicities. The most striking feature of the metal-free models is the high temperature they maintain at their photospheres. These stars derive their nuclear energy from a combination of inefficient proton-proton burning and CNO burning with the small fraction of C built up in the pre-MS phase (Castellani et al. 1983; El Eid et al. 1983). As a result of lower energy generation rates in the convective core, they maintain core temperatures in excess of 10^8 K to support the mass against gravitational collapse. Together with reduced radiative opacity in their envelopes, the high core temperatures of Pop III stars make these stars hotter and smaller than their metal-enriched counterparts. Using homology relations, and assuming constant (electron scattering) opacity and a CNO burning rate $\epsilon \propto \rho T_c^{12} (Z_{\text{CN}}/Z_{\odot})$, we find that massive stars have $R \propto (Z_{\text{CN}}/Z_{\odot})^{1/12}$ and $T_{\text{eff}} \propto (Z_{\text{CN}}/Z_{\odot})^{-1/24}$, in good agreement with our numerical models.

With g and T_{eff} in hand, we need a model atmosphere to derive the spectral luminosity distribution L_{ν} . A model atmosphere is necessary because a simple blackbody curve for each T_{eff} will not accurately reproduce the spectrum near the ionization edges of H I, He I, and He II. For each stellar model and $\log g - T_{\text{eff}}$ pair, we calculated a static, non-LTE model atmosphere that was used to predict the spectral luminosity distribution for the star. We used the atmosphere code TLUSTY (Hubeny & Lanz 1995) to produce the model atmospheres and its included package SYNSPEC to produce the continuum spectra. Figure 2 illustrates the change in the spectral distribution of a $15 M_{\odot}$ star at $Z = 0.001$ ($T_{\text{eff}} = 36,000$ K) and $Z = 0$ ($T_{\text{eff}} = 63,000$ K). The key between the two spectra is the high T_{eff} of the $Z = 0$ model. A Pop II star of the same T_{eff} would exhibit a continuum shape similar to the Pop III model but attenuated by metal-line absorption. However, a Pop II star at $15 M_{\odot}$ is unlikely to reach $T_{\text{eff}} = 63,000$ K during its MS lifetime.

3. IONIZING PHOTON YIELDS

The model atmospheres were used to predict the rates of ionizing photon production, Q_i , in photons s^{-1} , for all the modeled stars:

$$Q_i = 4\pi R_*^2 \int_{\nu_i}^{\infty} \frac{F_\nu}{h\nu} d\nu, \quad (1)$$

where R_* is the radius of the star, F_ν is the spectral flux distribution in $\text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1}$, and the indices $i = 0, 1$ and 2 correspond to integration over the H I, He I, and He II ionizing continua, $h\nu \geq h\nu_i$ (13.60, 24.58, and 54.40 eV, respectively). The Q_i for Pop III stars appear in Figure 3. Individual metal-free stars emit far more of their energy in photons with $h\nu > 13.6$ eV than do their Pop I and Pop II counterparts. This increase produces a 50% gain in the total ionizing photon production at high mass ($40 - 70 M_\odot$) and gains by factors of $2 - 40$ at moderate mass ($10 - 30 M_\odot$). Above $70 M_\odot$, a larger fraction of the star’s energy is released above 1 Ryd, but the average photon energy increases such that the overall gain in Q_0 is modest.

A striking feature of the ionizing photon production is the high fraction of the photons emitted with energy sufficient to ionize He I and He II. These fractions are expressed by the ratios Q_1/Q_0 and Q_2/Q_0 , displayed in Figure 3. High-mass Pop III stars emit 60 – 70% of their ionizing photons in the He I continuum and up to 12% of these photons in the He II continuum. For comparison, $Q_1/Q_0 \simeq 0.2 - 0.4$ for Pop I O3 – O5 stars with T_{eff} in the range 45,000 – 51,000 K (Vacca, Garmany, & Shull 1996; Schaerer & de Koter 1997).

The overall gain in integrated ionizing photon production is best evaluated with synthetic spectra of stellar clusters (Sutherland & Shull 1999; Leitherer et al. 1999), which compare the rates Q_i per unit mass of stellar material. Synthetic spectra of Pop III and Pop II zero-age clusters with a standard initial mass function (Salpeter IMF with $0.1 \leq M/M_\odot \leq 100$) appear in Figure 4. The gain in Q_0 (s^{-1}) is near 50% for this IMF. However, $\log Q_1$ increases from 52.4 to 52.9, and $\log Q_2$ increases from 46.1 to 51.7. This dramatic increase in the capability of stars to ionize He I and He II are not predicted by populations that approximate Pop III with existing metal-poor models.

4. IMPLICATIONS FOR REIONIZATION

Metal-free stars emit 50% more ionizing radiation per unit mass of stellar material than metal-enriched stars. Given the uncertain efficiency of star formation out of primeval gas (Abel et al. 1998), our conclusion that Pop III stars are more efficient ionizing sources (per unit mass) does not necessarily mean that they are more capable than other stellar populations of reionizing the universe. For this reason, we calculate a star formation rate (SFR) for metal-free stars necessary to reionize the universe and test this quantity against observations.

Madau, Haardt, & Rees (1999) imposed the condition that the universe is reionized when the number of ionizing photons emitted in one recombination time equals the mean number of

hydrogen atoms. Accounting for clumping of the IGM and assuming a typical rate for ionizing photon production per unit mass, they found that the critical SFR required to reionize by $z = 5$ is $0.013 f_{esc}^{-1} M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3}$, where f_{esc} is the fraction of ionizing photons that escape into the IGM (see Tumlinson et al. 1999 for a review). Based on the per-mass photon emission rates of the Pop III cluster in Figure 4, we estimate the critical rate of metal-free star formation to be $0.008 f_{esc}^{-1} M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3}$. This requirement would be comparable to the inferred (highly uncertain) SFR at $z = 5$ if $f_{esc} = 0.40$ (Madau, Mozetti, & Dickenson 1998).

In addition to enhanced ionizing capability, Pop III stars convert less of their initial mass into metals. Preliminary evaluation of the metal yields of Pop III stars (Woosley & Weaver 1995) indicates that for every solar mass of stars formed, $0.007 M_{\odot}$ in metals are released. Assuming that this value holds throughout the Pop III epoch, then the critical SFR for H reionization by Pop III stars implies a metal enrichment rate of $10^{-4} M_{\odot} \text{ yr}^{-1} \text{ Mpc}^{-3}$. Over the 500 Myr between $z = 10$ and $z = 5$, Pop III stars would enrich the universe to a mean metallicity $\langle Z \rangle \sim 6 \times 10^{-4} Z_{\odot}$ ($H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_b = 0.08$), 20% of the minimum value $Z = 10^{-2.5} Z_{\odot}$ observed at $z \sim 3$ and $\sim 2\%$ of the average metallicity in damped Ly α systems at that redshift (Pettini 1999).

Our new models of metal-free stars raise the possibility that stars are also responsible for He II reionization. Various studies (Reimers et al. 1997; Heap et al. 1999; Hogan, Anderson, & Rugers 1997) constrain the epoch of He II reionization to $z \sim 3$ with observations of patchy He II absorption. Conventional wisdom (Fardal et al. 1998; Madau et al. 1999) states that only the hard spectra of AGN could produce enough photons with $h\nu \geq 54.4 \text{ eV}$ to reionize He II, based in part on the result $Q_2/Q_0 \leq 0.02$ for low-metallicity stellar spectra to which Wolf-Rayet stars contribute only minimally (Leitherer & Heckman 1995). However, for the Pop III cluster in Figure 4, $Q_2/Q_0 = 0.05$. Thus, for $n_{He}/n_H = 0.0785$ ($Y = 0.239$) and assuming full ionization of H and He, the He III region excited by this cluster will have 50% the radius of its H II Strömgren sphere. This large He III region may imply that He II reionization differs in the denser regions, compared to the low-density IGM where recombinations are not important.

5. COSMOLOGICAL IMPLICATIONS

Pop III stars can be distinguished from metal-enriched stars by their theoretical radii and effective temperatures. Unfortunately, these features are not observable directly. However, these characteristics modify the spectrum of the stars in ways that are potentially observable. We discuss these in brief below, but defer detailed studies of metal-free stars to future work.

If the ionizing spectra of clusters follow a power law, $L_{\nu} \propto \nu^{-\alpha}$, we can use α to characterize the effects of Pop III stars on the intergalactic radiation field. The Pop III cluster in Figure 4 has a hard (and inverted) spectral index $\alpha_1 = -1.2$ in the range $1 - 4 \text{ Ryd}$ and $\alpha_4 = 2.0$ for $h\nu \geq 4 \text{ Ryd}$. For comparison, the Pop II cluster in Figure 4 has $\alpha_1 = 1.0$ and negligible flux above 4 Ryd . Thus, metal-free stars could contribute harder radiation to the extragalactic spectrum than

QSOs with intrinsic $\alpha_s = 1.8$ between $0.9 - 2.6$ Ryd (Zheng et al. 1997). At $z \sim 5$ this hard EUV spectrum is accessible to ground- and space-based instruments in the optical and near-UV.

The dramatic increase in He II ionizing photons from metal-free stars implies that they will have distinctive effects on their neighborhoods. Simple models of Pop III H II regions show that Pop III stars excite sizable He III regions. The $\lambda 1640$ and $\lambda 4686$ recombination lines of He II might be detected for targets with $z = 5 - 10$ in the $1 - 5 \mu\text{m}$ range with NGST. Similarly, the radio recombination lines of He II may provide a unique signature of these stars. However, He II recombination emission observed in the spectra of metal-poor extragalactic H II regions has often been attributed to Wolf-Rayet stars, X-ray binaries, or shocks. (Garnett et al. 1991; de Mello et al. 1998; Izotov et al. 1997). These sources are likely to be less important at high z , but they may complicate the identification of metal-free stellar populations.

The photodissociation of H_2 by FUV radiation ($912 - 1126 \text{ \AA}$) from the first luminous sources in the universe is suggested to have inhibited subsequent star formation near these sources by destroying their only coolant (Haiman, Rees, & Loeb 1997; Ciardi, Ferrara, & Abel 1999). Photons with $h\nu = 11.2 - 13.6 \text{ eV}$ can propagate freely into neutral, dust-free gas and dissociate H_2 by exciting permitted transitions to the $^1B\Sigma_u^+$ state, 10 – 15% of which decay into the continuum. This “negative feedback” is presumed to precede the ionization front by a distance dependent on the spectrum of the ionizing source. Pop III clusters, with enhanced ionizing photon production and suppressed FUV flux, may dissociate H_2 with their ionization fronts.

The hard spectra of Pop III stars may also affect the IGM ionization ratios through changes to the extragalactic spectrum. Fardal et al. (1998) define the ratio $\eta \equiv N_{\text{HeII}}/N_{\text{HI}}$ to express the relative column densities of He II and H I. This ratio is sensitive to the shape of the extragalactic spectrum and can be used to predict the optical depths τ_{HI} and τ_{HeII} in the IGM. An increase in the He II ionizing flux favors He III and decreases η . Fardal et al. (1998) estimate that $\eta \gtrsim 100$ is necessary to explain $\tau_{\text{HeII}} \sim 1 - 5$ observed at $z \sim 3$ (Davidsen, Kriss, & Zheng 1996). The $Z = 0$ stellar spectra give $\eta \sim 30$, inconsistent with the observed opacity if these stars are still forming at that epoch. Thus, $Z = 0$ stars may not dominate the intergalactic radiation field at $z = 3$.

In summary, we outline a general picture of the era of Pop III stars based on our models and consistent with the observations discussed above. We assume that the first stars formed at $z \sim 10$, consistent with simulations of large-scale structure. If the total cosmic star formation rate exceeded the critical rate calculated in § 4, then these first stars may have reionized hydrogen and helium in the universe. Upon their deaths, they enriched the universe to an average metallicity 20% of that observed at $z \sim 3$. Population III then faded, leaving their metal-enriched progeny to provide the rest of the metals and the softer radiation seen at $z \sim 3$.

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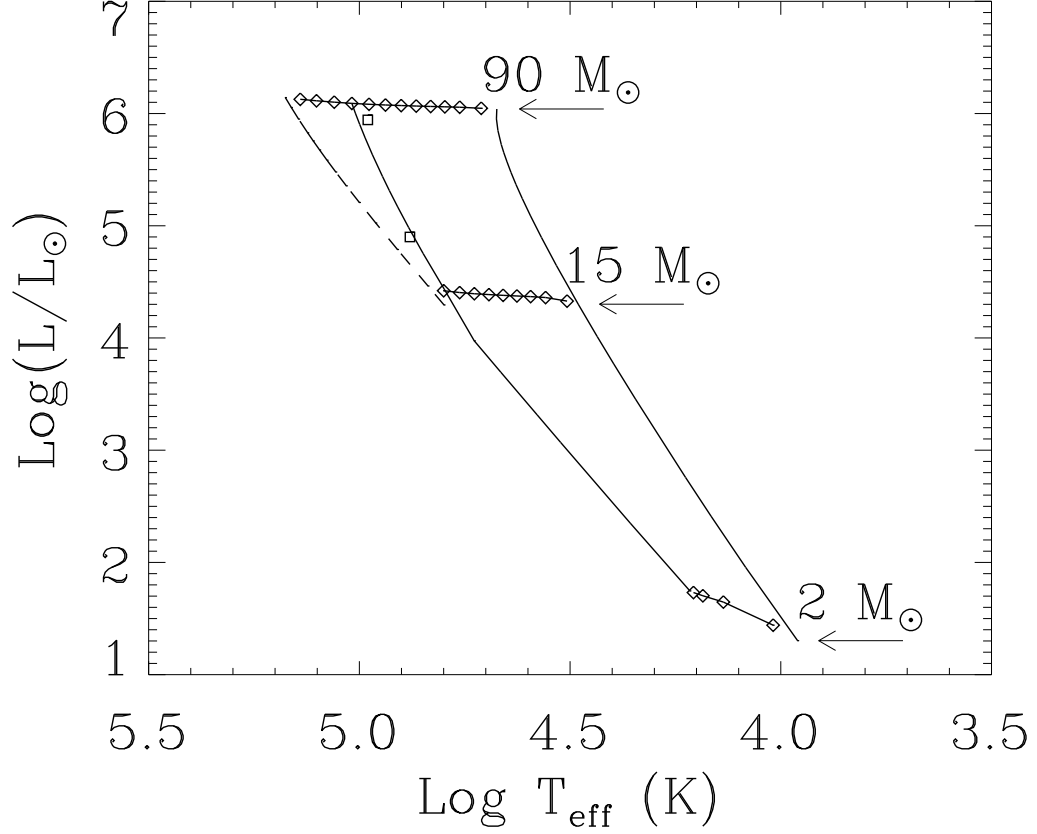


Fig. 1.— Zero-age main sequences for Pop I ($Z_{\odot} = 0.02$) and Pop III stars of mass 2, 5, 8, 10, 15, 20, 25, 30, 35, 40, 50, 60, 70, 80, and $90 M_{\odot}$. The diamonds mark decades in metallicity in the approach to $Z = 0$ from 10^{-2} down to 10^{-5} at $2 M_{\odot}$, down to 10^{-10} at $15 M_{\odot}$, and down to 10^{-13} at $90 M_{\odot}$. The dashed line along the Pop III ZAMS assumes pure H-He composition, while the solid line marks the upper MS with $Z_C = 10^{-10}$ for the $M \geq 15 M_{\odot}$ models. Squares mark the points corresponding to pre-enriched evolutionary models from El Eid et al. (1983) at $80 M_{\odot}$ and from Castellani et al. (1983) for $25 M_{\odot}$.

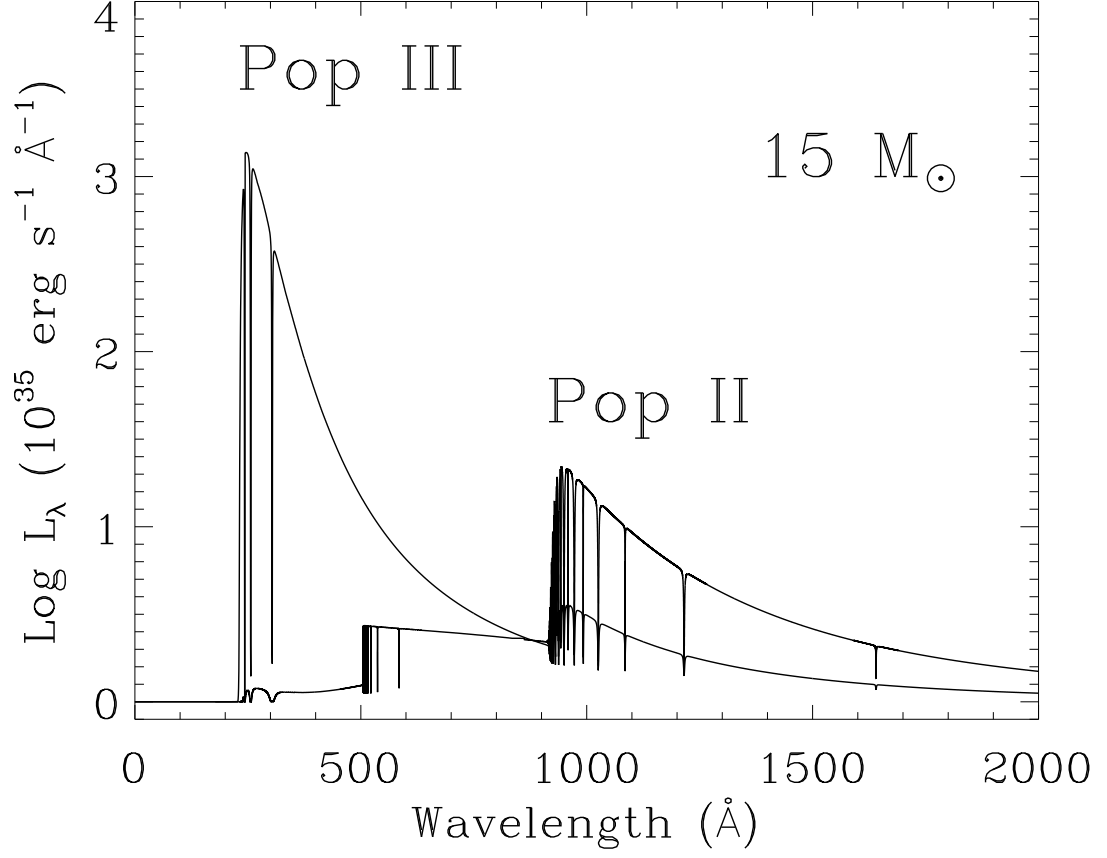


Fig. 2.— Comparison of spectra calculated from atmosphere models of Pop II ($Z = 0.001$) and Pop III ($Z = 0$) stars of $15 M_{\odot}$. The Pop II star has $T_{\text{eff}} = 36,000$ K, and the Pop III star has $T_{\text{eff}} = 63,000$ K. Only H I, He I, and He II lines are included.

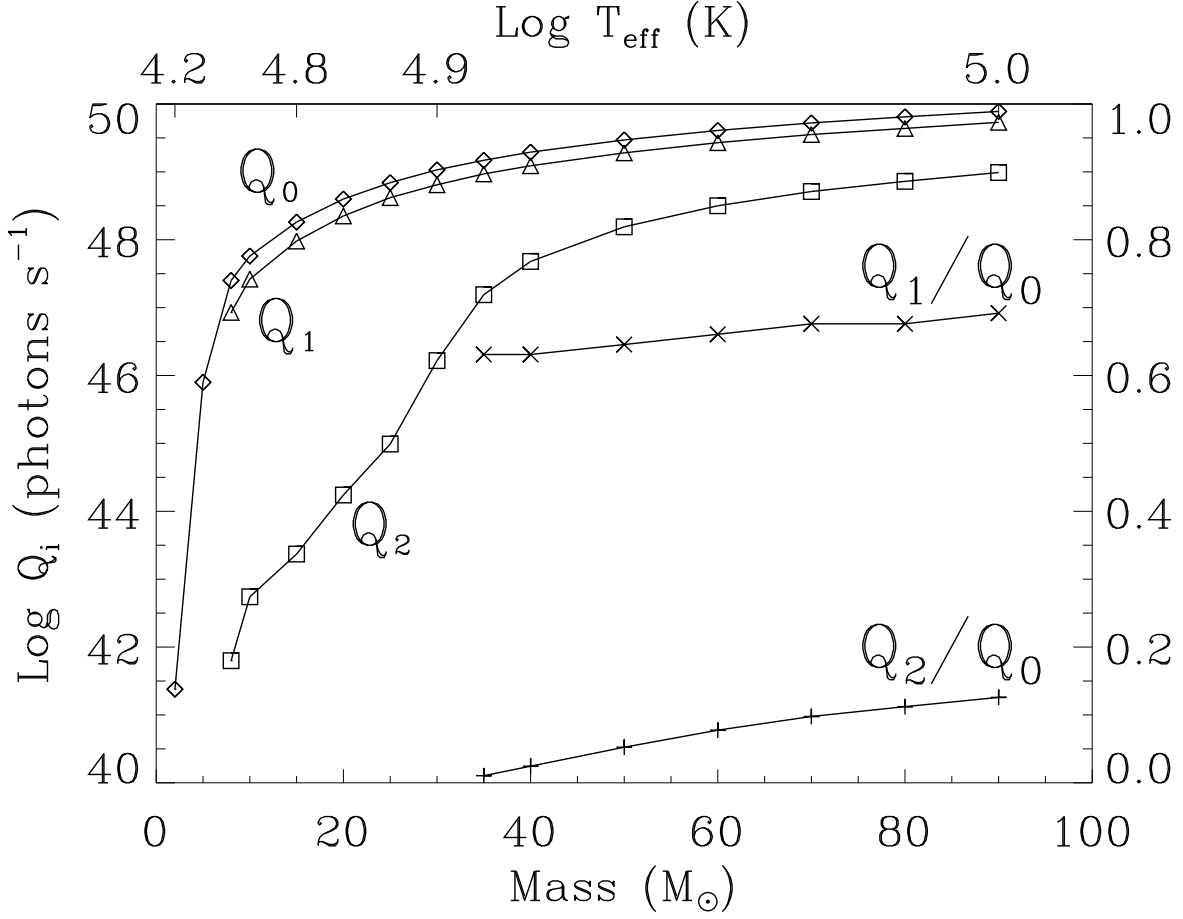


Fig. 3.— Ionizing photon yields for initially metal-free stars pre-enriched to $Z_C = 10^{-10}$. High-mass ($M > 25 M_{\odot}$) stars emit 60 – 70% of their ionizing photons in the He I ionizing continuum and 2 – 12% in the He II continuum. The right axis corresponds to the lower two curves, which represent the ratios Q_1/Q_0 and Q_2/Q_0 of ionizing photons emitted in the He I and He II continua. The top axis is $\text{log } T_{\text{eff}}$ for the displayed models.

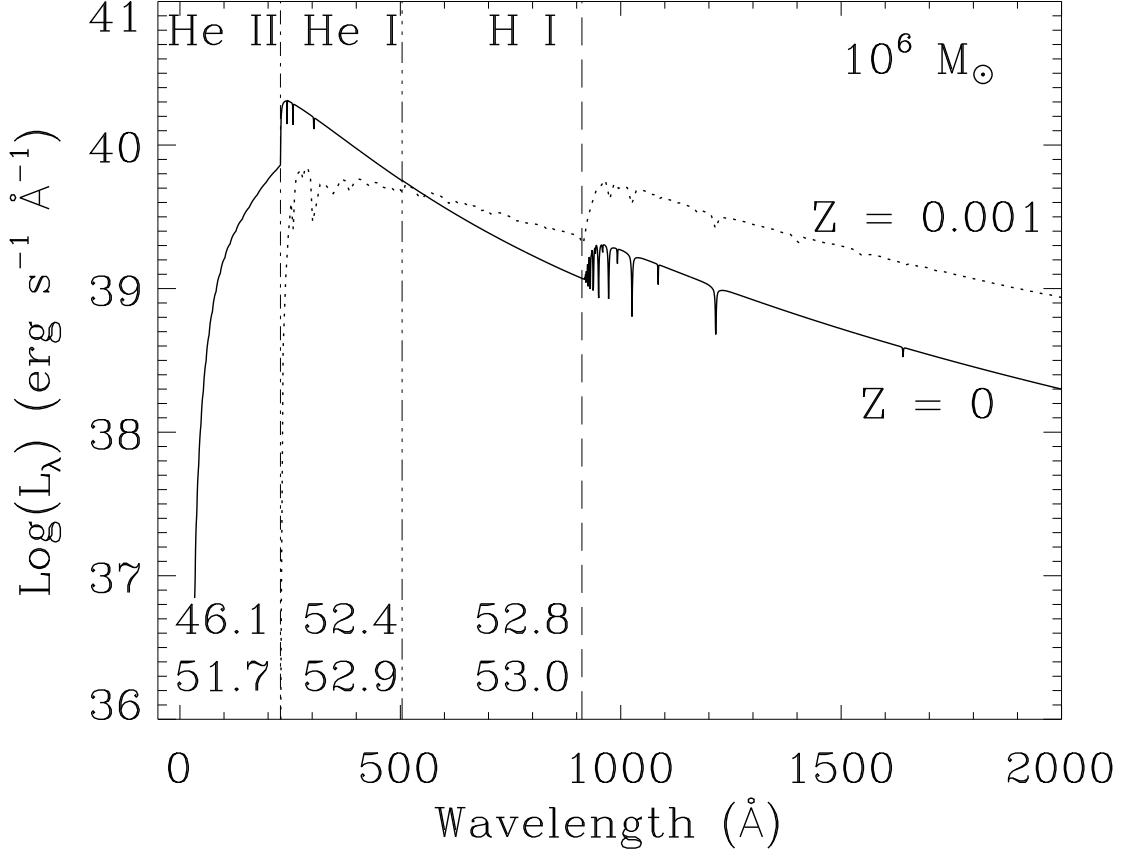


Fig. 4.— Synthetic spectra of Pop II and Pop III clusters of $10^6 M_\odot$ with a Salpeter IMF. The Pop II spectrum ($Z = 0.001$) represents an instantaneous burst of star formation at an age of 1 Myr and is from Leitherer et al. (1999). The Pop III cluster was constructed with the pre-enriched MS displayed in Figure 1. The dashed lines mark the three continuum transitions of H I, He I, and He II, from right to left respectively. The numbers in the lower left, near each continuum mark, represent $\log Q_i$ for that continuum, with Pop II given above.